03B_Thermodynamic Approach to Fracture

Fracture is the propagation of a pre-existing flaw (also called a micro-crack) under and applied stress. Therefore, the critical condition for crack propagation must coalesce the applied stress with the flaw size. This is the field of Fracture Mechanics. It developed rapidly in analytical form in the sixties. Later on, and even today, it has become more computational.

Our approach is to take a fundamental approach so as to grasp the basic principles that form the foundation of Fracture Mechanics. This approach combines thermodynamics and mechanics.

The thermodynamic approach, first developed by Griffith, is described here.

It is based upon the work required to propagate the crack and the work that can be done my mechanical forces and displacement on the system to overcome the work of fracture. These two contributions to the overall thermodynamic analysis are summarized in the following Table.

Local Work on the System	**Work Done by the Surroundings**	
Work of Fracture when the crack grows $2\gamma_F$ Units Jm^{-2} when multiplied by crack advance: 2D problem = $(2\gamma_F)1 * \delta c$ Penny Shaped Crack = $(2\gamma_F)d(\pi c^2) = (2\gamma_F)2\pi c\delta c$ The work has units of Joules	Reduction in Stored Elastic Energy	Reduction in Potential Energy (gravitational energy in a load)
	$\frac{\sigma^2}{2E}(effective\ volume)$ Units J	Load x Displacement $P.\delta u$ Units of J

The method of analysis is to consider a crack placed under certain stresses and forces to advance by an incremental amount. The work done to advance the crack in this way is compared to the work done by the mechanical system when the crack moves by a small distance.

Remember that applied stresses and forces can produce a driving force to advance the crack only is there is a give-and-take relationship between the movement of the crack and physical displacements in the mechanical system. Keep in mind that work is a product of force and displacement, therefore, crack advance must be accompanied by discernible displacements in the system.

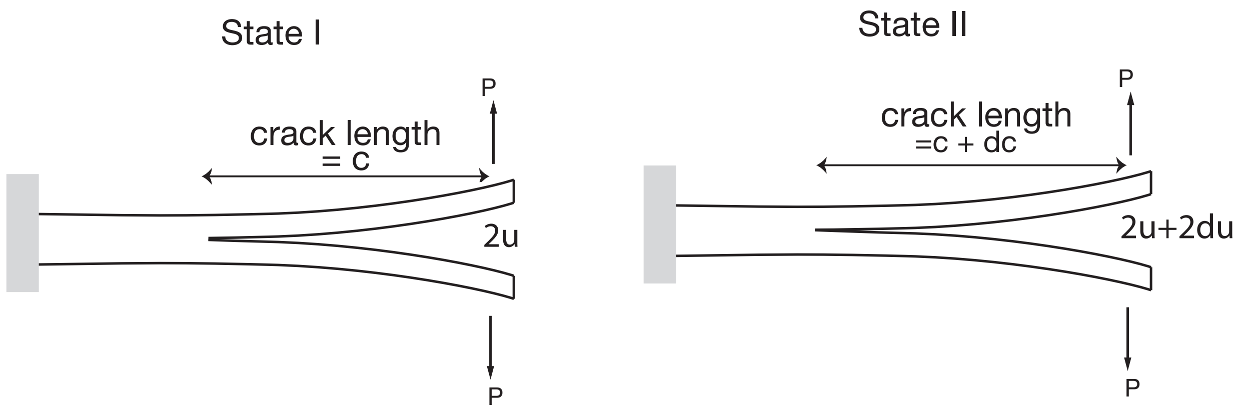
The left-hand column in the Table, above, describes the work of fracture for an incremental growth of a crack from $c \rightarrow (c + \delta c)$. Two cases are considered, a ribbon-shaped edge-crack with a unit depth normal to the paper (the 2D case) and a penny shaped crack in the middle of the sample (the 3D case).

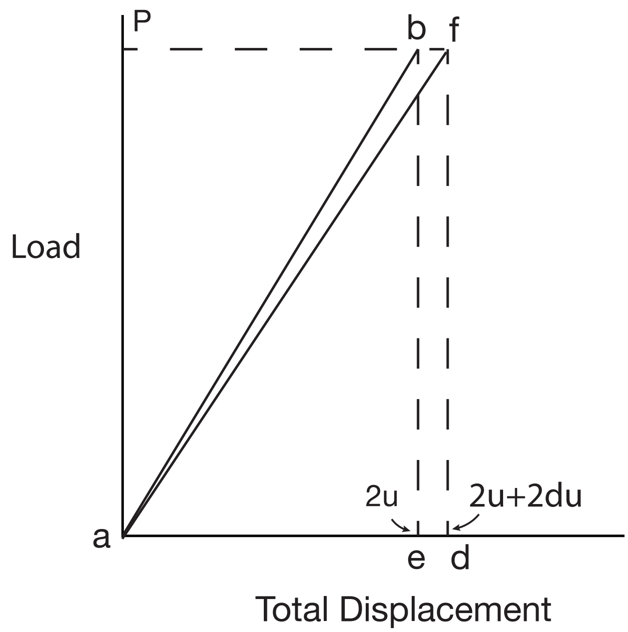
$2\gamma_F$ is the work done when the crack advances. Since the work of fracture is expected to be proportional to the advance of the crack, that is, to the creation of new surfaces, the work of fracture can be expressed as a pseudo-surface energy. Hence it is written as $2\gamma_F$ in units of J m^{-2}. The factor of 2 arises from the generation of two surfaces during fracture.

The two sub-columns on the right describe sources of mechanical work done by the system to advance the crack. One of them is the change in the elastic energy in the system, and the second is the change in the potential energy (imagine a load applied to the specimen - as the crack advances the sample can become more compliant so that the load would drop - this work will be available to provide the work of fracture).

As an example let us apply this method to fracture in a double cantilever beam specimen, as shown in the following section.

Fracture Analysis for a DCB (Double Cantilever Beam Specimen)





Note: The specimen has unit thickness normal to the plane of the paper. Therefore, the load P has units of Nm^{-1} (not just Newtons)

The Method

The method consists of comparing the mechanical and fracture energy in State II with State I. If the difference is negative then the crack will propagate. Setting the different to zero therefore, gives the condition for fracture. These two parts of the energy equation are described below.

The Work of Fracture

State II - State I: $$2\gamma_F(c+\delta c)-2\gamma_F c = 2\gamma_F \delta c \qquad (1)$$

Mechanical Work

(i) Change in Potential Energy (II -I) =

(ii) Change in the stored elastic energy (II -I)= (Area Triangle *afd*) - (Area Triangle *abe*)

= +(Area Triangle *abf*)

From geometry we note that

(i) + (ii) = $-P2\delta u$ +(Area Triangle *abf*) = -(Rectangle *ebfd*) +(Area Triangle *abf*) (2)

Since the rectangle area is exactly twice the area of triangle, the total mechanical work may be written as either

$$-\frac{P2\delta u}{2}, \text{ or as } -\left(\text{Area Triangle } abf\right) \qquad (3)$$

That is (and this is important)

The total change in mechanical work (State II - State I) may be written to be equal to (both with a negative sign)

•one half of the change in the potential energy

or, as

•the increase in the stored elastic energy

we will use the above result later in analysis of a penny shaped crack.

The Fracture Criterion

The fracture criterion is obtained by adding the work of fracture in Eq. (1) with the change in elastic energy, Eq. (2) and equating the sum to zero

$$+2\gamma_F\delta c - \frac{P2\delta u}{2} = 0$$

$$\frac{du}{dc} = \frac{2\gamma_F}{P} \qquad (4)$$

Note that the increase in the stored elastic energy has been folded into the change in the potential energy as discussed just above.

Noting that $S = \frac{2u}{P}$; $u = \frac{P}{2}S$

so that $\frac{du}{dc} = \frac{P}{2}\frac{dS}{dc}$ (5)

Combining (4) and (5), we obtain

$$2\gamma_F = \frac{P^2}{2}\frac{dS}{dc} \qquad (6)$$

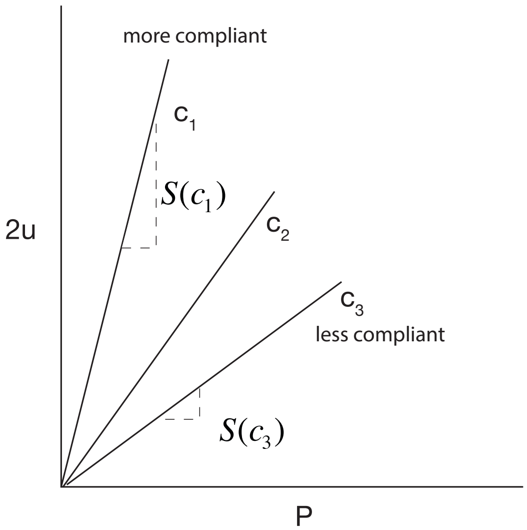
Notes:

where,

P is the load (per unit depth of the crack perpendicular to the plane of the paper) for the onset of fracture

and $\frac{dS}{dc}$ is the change in compliance with crack length.

Both of the above parameters are experimentally accessible. Therefore Eq. (6) can be used to measure the work of fracture, which is a material parameter.



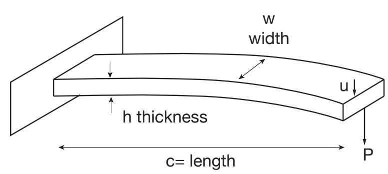
To measure $\frac{dS}{dc}$ we make DCB (double cantilever beam) specimens with different crack lengths, and measure the compliance with the application of small loads (less that those that may cause fracture) which will give plots such as

The quantity $\frac{dS}{dc}$ is then obtained by a plot of compliance versus the crack length.

In this way, all parameters in Eq. (6) are experimentally determined, leading to the measurement of the work of fracture $2\gamma_F$.

Equation for Beam Deflection

The equation for the deflection of a beam under a point load at its end is given by



$$u = \frac{Pc^3}{3EI}, \quad I = \frac{wh^3}{12} \qquad k = \frac{P}{2u}, \quad S = \frac{2u}{P}$$

The equations for beam deflection give above can be used to calculate $\frac{dS}{dc}$ without having to the measure this quantity experimentally.

Q: If $2\gamma_F$ is a material parameter then can it be found in a handbook of material properties?

The answer is yes, but in the form of the fracture toughness, K_{IC}, which is related to the work of fracture by

$$2\gamma_F = \frac{K_{IC}^2}{E}, \qquad (7)$$

where E is the Young's modulus. In Eq. (7) all quantities are material parameters. Handbooks give the values for fracture toughness rather than the work of fracture because the toughness is used in engineering design.

Fracture toughness, as we shall see in the next section has units of MPa $m^{1/2}$.

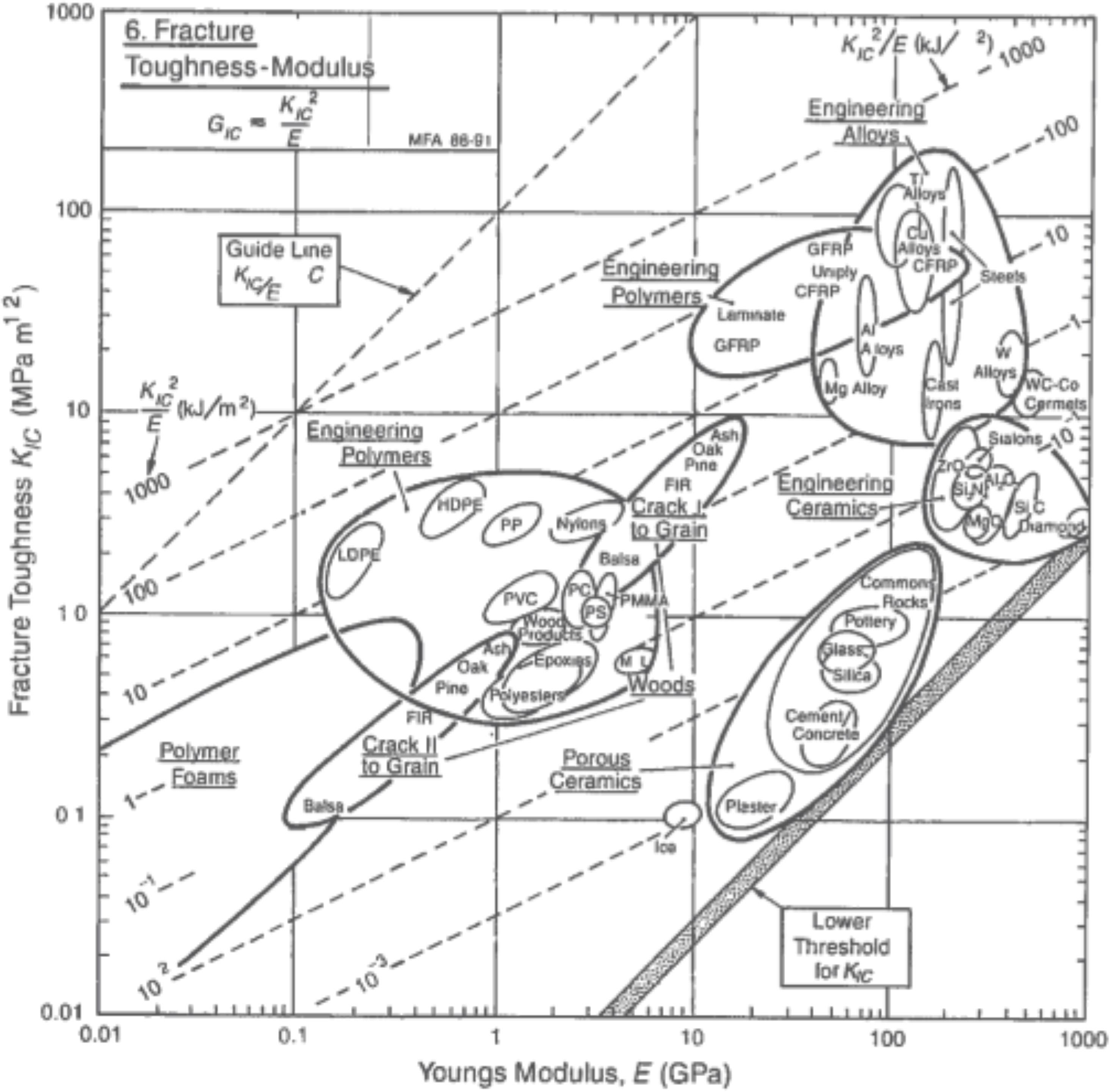
The right-hand side of Eq. (7) has units of $(MPa)^2$m/MPa, or (MPa)m

Pa has units of energy per unit volume. Nm^{-2} is the same as $(N*m)*m^{-3}$, that is J m^{-3} (energy per unit volume).

Therefore MPa*m has units of energy per unit area which are the units of $2\gamma_F$.

We shall derive Eq. (7) in the following section, but let us first discuss the values for the parameters in Eq. (7) for different classes of materials.

The following figure shows a "map" of the fracture toughness vs. the Youngs Modulus, E for different classes of materials that includes polymers, ceramics, metals and graphite-fiber-polymer composites.



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Notes:

•The dotted lines are for constant values of $2\gamma_F$ $(=\frac{K_{IC}^2}{E})$